# 2

# Fundamentals of Wave Optics

**Summary.** Chapter 2 discusses the basics of wave optics and briefly introduces the reader into the wave propagation in an optical medium, which simplifies the study of the material presented in following chapters. Such terms as optical medium and its theoretical description are described shortly. The Maxwell's equations are written and principal conception of plane wave propagation is discussed. Then, optical wave behavior in a dielectric medium is considered by means of general solution of the wave equation and such crucial parameters as polarization, refractive index and group velocity are considered.

## 2.1 What is an Optical Medium? – Theoretical Models

The interaction between electromagnetic field and medium is the main question for understanding of fundamentals of optical wave propagation in waveguides and operation of many other passive and active optical devices.

Discussion of the optical wave fundamentals should consist of the following questions: what optical medium is; what properties the medium has; how optical medium influences electromagnetic field; and, at last, how to describe the electromagnetic field and its interaction with optical medium. Generally, that problem is illustrated in Fig. 2.1 where the spherical electromagnetic wave irradiated from the point source incidents the object. To solve the problem of their interaction, we should characterize the field inside and outside the medium placed in the free space, taking into account its modification at the boundary of the object.

The distances between atoms are one of the key points in characterization of interaction with light. In optical devices and optical structures the thickness of layers in case of layered structure or elements size in case of PhC can be comparable with the wavelength. In such a case, interaction of light with the medium may have different effects like transparency, total or partial reflections and refraction. In optical media used in lightwave technology, the distance between atoms is of the order of 0.1 nm, which is small compared to the wavelength of light that is used in optical communications (these

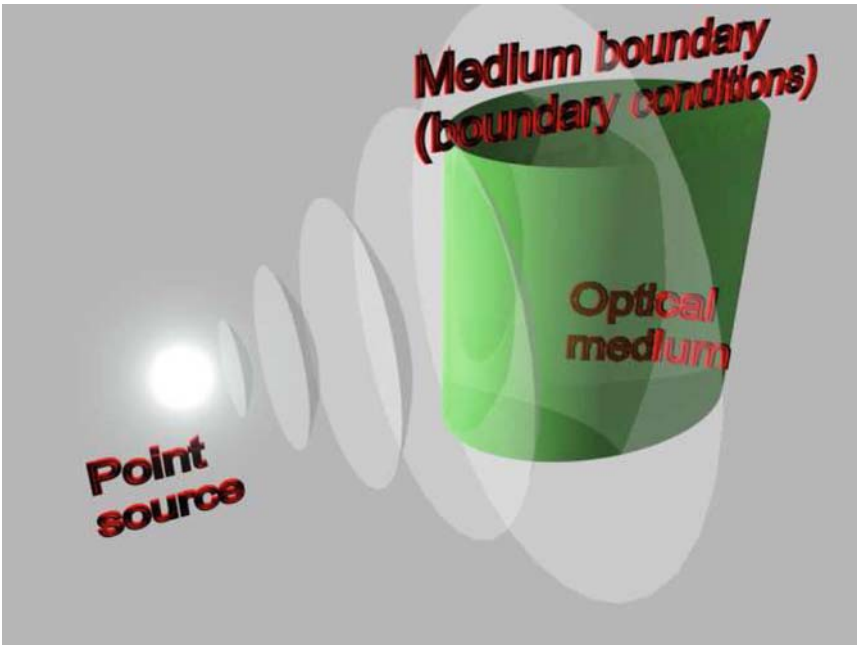


**Fig. 2.1.** Problem of interaction between EM field radiated from the point source and optical medium

wavelengths are in range of 0.8–1.6 μm). With this, the medium is assumed to be a homogeneous medium. Moreover, in many cases, optical medium can be assumed to be isotropic and time invariant. Hereafter, it will be assumed that the properties of an optical medium are described by the permittivity $\varepsilon$, the permeability $\mu$ and the conductivity $\sigma$.

In special cases, an optical medium has nonlinear response to an external influence. Nonlinear effects like second harmonic generation, Kerr effect, solitons and vortices formation, etc., plays a more important role in advanced optoelectronics and photonics.

### 2.1.1 Wave Optics

When $\varepsilon$, $\mu$ and $\sigma$ for the material are defined, then Maxwell's equations solution is a good starting point for analyzing the propagation of light also known as the wave optics. The wave optics provides, particularly, the problem solution of electromagnetic waves propagation in optical waveguides, namely, determination of the electric and magnetic components' amplitudes of the optical field, as well as its phase and amplitude distribution in space. If the thickness of the core of the optical waveguide is of the order of the wavelength, then the propagation can be described with a few modes which are functions of the optical waveguide parameters and the wavelength of the light. If, on

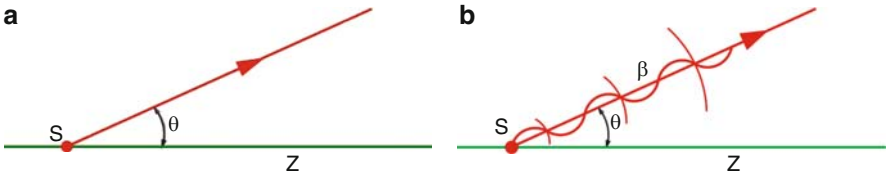


**Fig. 2.2.** Optical radiation representation in the geometrical (**a**) and wave (**b**) optics

the other hand, the core radius is large as compared to the wavelength, then many propagating modes appear. In this case, it will be more effective to solve the problem by means of geometric optics.

### 2.1.2 Geometric Optics

When dimensions of objects are large compared to the wavelength of the light, an approximate method can be used to study the light propagation. The geometric optics or ray optics uses the methods of geometry in order to formulate the laws of optics. In geometric optics the concept of light rays is introduced in order to describe optical phenomena. The paths taken by the light rays in heterogeneous and compound media are derived from the so-called eikonal equation [201].

Figure 2.2 schematically shows correspondence of both methods. The light irradiated from the point source can be represented like a ray (light beam shown on the Fig. 2.2a) directed angularly from the point source with the angle $\theta$ to the optical axes $z$. This angle $\theta$ corresponds to the propagation vector $\beta$, which has the same angle to the axis (see Fig. 2.2b) and characterizes the direction of wave propagation with the same angle $\theta$ and its phase. The propagation vector is perpendicular to the constant phase surfaces of the propagating wave which are marked by concentric arcs with the center in the point S.

## 2.2 Wave Propagation: Maxwell's Equations

Light consists of an electric field and magnetic field that oscillate at very high rates, of the order of $10^{14}$ Hz. This field propagation is described by periodical functions. Transfer of electromagnetic wave energy through empty space is realized by electric and magnetic fields which exchange the energy obeying Ampere's and Faraday's laws. Magnetic field variation is perpendicular to the electric field. A single frequency electromagnetic wave exhibits harmonic variation of electric and magnetic fields in space. In any fixed location, the field amplitude varies with optic frequency. The magnitude of the field repeats itself after one period of the oscillation. The wave repeats itself in space after a distance $\lambda$ called wavelength which is actually spatial period of the wave. The reciprocal quantity is called wave number.

Within the classic theory, all the electromagnetic phenomena are described by the system of Maxwell's equations which gives the time dependence of electric and magnetic fields and their interaction with optical medium.

Before discussing light propagation in complex optical structures such as PhCs, we consider its propagation in free space. For a medium with free charges and with current, Maxwell's equations are taken in the following form

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{2.1}$$

$$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \tag{2.2}$$

$$\nabla \cdot \mathbf{D} = \rho, \tag{2.3}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2.4}$$

where $t$ denotes the time, $\boldsymbol{\nabla}$ is the nabla operator, "$\cdot$" and "$\times$" denotes scalar and vector products. In Cartesian coordinates,

$$\boldsymbol{\nabla} = \mathbf{x}\frac{\partial}{\partial x} + \mathbf{y}\frac{\partial}{\partial y} + \mathbf{z}\frac{\partial}{\partial z},$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are unit vectors. The vectors $\mathbf{E}$ and $\mathbf{H}$ are the coordinate- and time-dependent vectors of electric and magnetic fields, respectively, $\mathbf{D}$ and $\mathbf{B}$ are corresponding dielectric displacement and magnetic inductance. The sources of the electromagnetic field are the charge density $\rho$ and the current density $\mathbf{J}$. They are connected by the continuity equation, which is given by taking divergence of (2.2) and using (2.3) and the relation $\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \mathbf{H} = 0$:

$$\boldsymbol{\nabla} \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}.$$

In case of nonconductive medium like silica or other material used for optical waveguiding in passive devices like optical fiber or planar waveguide, $\mathbf{J} = 0$ and $\rho = 0$. The flux densities are related to the field vectors by the material relations

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \tag{2.5}$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}, \tag{2.6}$$

where

$$\varepsilon_0 = 8.85 \cdot 10^{-12} - \left[\frac{As}{Vm}\right]$$

is the electric constant,

$$\mu_0 = 4 \cdot 10^{-7} \left[\frac{Vs}{Am}\right]$$

is the magnetic constant, $\mathbf{P}$ is the electric polarization of the medium induced by the electric field $\mathbf{E}$ in the medium and $\mathbf{M}$ is the magnetic polarization of

the medium. For optical fibers $\mathbf{M} = 0$ because of the nonmagnetic nature of silica glass. The product of constants $\varepsilon_0$ and $\mu_0$ is equal to

$$\varepsilon_0 \cdot \mu_0 = 1/c^2, \tag{2.7}$$

where $c \approx 3 \cdot 10^8\,\mathrm{m/s}$ is the light speed in vacuum. The concept of $\mathbf{P}$ will be explained below. The response of individual electric charges to an electric field can be described by Newton's laws. The behavior of a charged fluid has a field description that will not be discussed here. The behavior of many solids in response to an electric field can be described well with the same kind of approximation which are usually made for stress and strain. If we think of stress as the perturbation applied to a sample and the strain as the response of the sample to the perturbation, we can say that in our model the response is proportional to the perturbation. The response of many common solids to an applied electric field is such that the electric field inside the sample is less than the applied one. It is similar to the solid that has produced its own charge distribution, i.e., becomes polarized, so that it produces an electric field which is added in opposition to the applied field. Materials which possess such properties are called dielectrics.

We consider the polarization of the material as the response in this case. It turns out that in many useful situations the polarization is proportional to the applied electric field. We need to define the polarization in a way that allows us to measure it.

Evaluation of the electric polarization $\mathbf{P}$ requires a microscopic quantum mechanical approach. Although such an approach is essential when the optical frequency is near a medium resonance, a phenomenological relation between $\mathbf{E}$ and $\mathbf{P}$ can be used far from the medium resonance. This is the case for optical fibers at the wavelength region $0.5 - 2\,\mu\mathrm{m}$, a range that covers the low-loss region of optical fibers and is of interest for fiber-optic communication systems. In a homogeneous linear and isotropic dielectric medium, which are considered in the book, polarization is aligned with and proportional to the electric field $\mathbf{E}$. In an anisotropic material, the polarization and the field are not necessarily in the same direction. In general, the relation between $\mathbf{E}$ and $\mathbf{P}$ can be nonlinear. Although the nonlinear effects in optical waveguides deserve consideration according to [202], they can be ignored in a following description of fiber modes.

Electromagnetic waves carry energy as they travel through empty space. There is an energy density associated with both the electric and magnetic fields. The amount of energy transport per unit area is described by the vector

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}, \tag{2.8}$$

which is called the Poynting vector. This expression is a vector product, and since the magnetic field is perpendicular to the electric field, $\mathbf{S}$ will be perpendicular to $\mathbf{E}, \mathbf{B}$- plane and coincides with a wave propagation direction.

## 2.3 Wave Equation in Vacuum

In a vacuum, there is no medium. As a result, there is no induced polarization or current. In other words, both **P** and **J** are equal to zero. Therefore, the Maxwell's equations are reduced to

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{2.9}$$

$$\boldsymbol{\nabla} \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}, \tag{2.10}$$

$$\nabla \cdot \mathbf{D} = 0, \tag{2.11}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{2.12}$$

Four equations (2.1)–(2.4) describe the interdependence between **E** and **H**. To solve the set of equations (2.9)–(2.12) one can eliminate **H** and derive an equation for **E** only. Taking the curl of $\nabla \times \mathbf{J}$ and employing the material equation (2.6) gives

$$\nabla \times \nabla \times \mathbf{E} = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right) = -\varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2}\mathbf{E}, \tag{2.13}$$

since

$$\nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \mu_0 \mathbf{H}) = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) = -\varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2}\mathbf{E}.$$

Vector calculus gives the identity

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E}. \tag{2.14}$$

Here,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

is the Laplace operator and equation (2.5) was used which gives $\nabla \cdot \mathbf{E} = \nabla \cdot (\varepsilon_0 \mathbf{D}) = \varepsilon_0 \nabla \cdot \mathbf{D} = 0$. Therefore, the wave equation for the electric field in vacuum is

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = 0. \tag{2.15}$$

The similar wave equation can be obtained for magnetic component by eliminating **E** from equations (2.9)–(2.10). The general form of the solution of the Eq. (2.15) has the form:

$$E(x, y, z, t) = a_+ E^+(\mathbf{k} \cdot \mathbf{r} - \omega t) + a_- E^-(\mathbf{k} \cdot \mathbf{r} + \omega t), \tag{2.16}$$

where $\mathbf{r} = (x, y, z)$ is the coordinate vector, $\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector, $\omega = 2\pi/\lambda$ is the angular frequency and $\lambda$ is the wavelength. Functions $E^+$

and $E^-$ describe the wave behavior in space (argument $\mathbf{k}\cdot\mathbf{r}$) and time (argument $\omega t$), and $a_+$ and $a_-$ are amplitude coefficients depending on boundary conditions.

A wave vector is a vector which specifies the wavenumber and direction of propagation of the wave. The magnitude of the wave vector indicates the wavenumber. The orientation of the wave vector indicates the direction of wave propagation. Wavenumber indicates the number of electric and magnetic vectors oscillations per unit of space and is measured in $m^{-1}$. Components of the wave vector corresponds to the wave numbers in $x$, $y$ and $z$ directions as follows

$$\frac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2. \tag{2.17}$$

The physical meaning of the wave solution given by (2.16) can be understood as follows. First, let us consider the special case when $k_x = k_y = 0$. In this case the argument $(\mathbf{k}\cdot\mathbf{r} - \omega t)$ is reduced to

$$k_x r_x - \omega t = k_x(r_x - \frac{\omega}{k_x}t).$$

This means that the wave $E^+$ is a wave propagating in positive $\mathbf{k}$ direction at velocity of $\omega/k_x$. Similarly, $E_-$ represents a wave propagating in negative $\mathbf{k}$ direction. In general, when all components of the wave vector are non-zero, wave propagates in $\mathbf{k}$ direction with a velocity of light. The special case is important for understanding of wave propagation and for practical applications in wave equation solution (2.15) in case when the field has only one component. This solution is called the plane wave. This case is described as

$$E(x, y, z, t) = Ee^{j(\omega t - k_z \cdot z)}\mathbf{x}. \tag{2.18}$$

Here, the electric field has only one component in the $x$-direction and propagates in the $z$-direction. Figure 2.3 shows a plane sine wave in two dimensions. The large arrow is a wave vector, which defines the direction of wave propagation by its orientation perpendicular to the wave fronts. As in case of spherical wave shown on the Fig. 2.2 the wave fronts of the plane wave are constant phase plane surfaces separated by one wavelength. The evolution time of the wave is defined by a term $(\omega t)$. For a plane sine wave, the formula (2.18) is transformed to

$$E(x, y, z, t) = \sin(\mathbf{k}\cdot\mathbf{r} - \omega t) = \sin(k_x x + k_y y + k_z z - \omega t).$$

If we assume that a wave front is a line along the comb of the wave, then the wave fronts are lines or surfaces of constant phase, and the equation defining a wave front is simply $\mathbf{k}\cdot\mathbf{r} = \text{const}$. For one dimensional propagation of light just along the $x$ axis we obtain the scalar wave number

$$k_x = \frac{\omega}{c} = \frac{2\pi}{\lambda}. \tag{2.19}$$

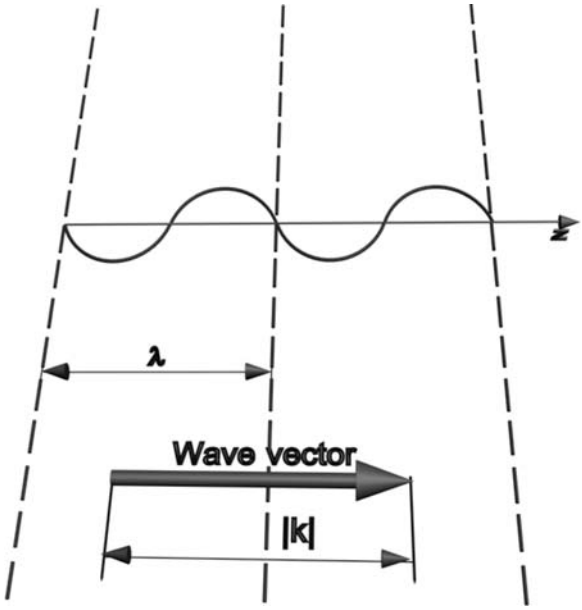


**Fig. 2.3.** Plane sine wave definition

Substituting electric field equation (2.18) into (2.9) and integrating over time, we obtain:

$$\mathbf{H} = \frac{k}{j\mu_0\omega} E_x e^{j(\omega t - k_z \cdot z)} \mathbf{y} = -j\sqrt{\frac{\varepsilon_0}{\mu_0}} E e^{j(\omega t - kz)} \mathbf{y}. \tag{2.20}$$

Thus, both the electric field and magnetic fields are perpendicular to the wave propagation.

## 2.4 Waves in Dielectric Media

In a dielectric medium, a nonzero polarization **P** is induced by an external electric field **E**. To understand the physical meaning of **P**, we consider a medium made of positive and negative charges (e.g., protons and electrons). When an electric field is present, it separates the positive and negative charges. This charge separation results in an additional electric field. This additional electric field is called the induced polarization.

Various media respond differently to a given external electric field. When the medium is linear and isotropic, **P** is linearly proportional to **E** and can be expressed as

$$\mathbf{P} = \varepsilon_0 \cdot \chi \mathbf{E}, \tag{2.21}$$

where $\chi$ is called the electric susceptibility. It is the proportion coefficient between polarization and electric field.

Linear susceptibility is, in general, a second-rank tensor but reduces to a scalar for an isotropic medium such as silica glass. Optical fibers become slightly birefringent because of unintentional variations in the core shape or in local strain. In general, the susceptibility is a function of the frequency $\omega$ of the applied field. When the field is an arbitrary function of time $t$, the polarization is a convolution of the Fourier transform of $\chi(\omega)$ with the $\mathbf{E}(t)$. This reflects the fact that the dipoles in the material cannot respond instantaneously to the applied field, and causality considerations lead to the Kramers–Kroenig relations.

With this definition, the electric displacement is

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \varepsilon \mathbf{E}, \tag{2.22}$$

where

$$\varepsilon = 1 + \chi \tag{2.23}$$

is called the dielectric constant or the relative permittivity of a dielectric material. The dielectric constant of a material affects how electromagnetic signals (light, millimeter-waves, etc.) move through the material. A high value of dielectric constant makes the distance inside the material look bigger. This means that light travels more slowly. It also compresses the waves to behave as if the signal had a shorter wavelength.

### 2.4.1 Refractive Index of Dielectric Medium

For an electromagnetic field in a dielectric medium without current, Eq. (2.10) reduces using (2.22) to

$$\nabla \times \mathbf{H} = \varepsilon \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{2.24}$$

Using the same procedure described earlier for Eq. (2.15), the wave equation for the electric field is

$$\nabla^2 - \frac{\varepsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0. \tag{2.25}$$

Therefore, the only difference of this equation from (2.15) is that the wave propagation velocity changes from $c$ to $c/n$, where $n = \sqrt{\varepsilon}$ is the refractive index of the medium. Consequently, the refractive index of a medium is a measure for how much the speed of light (or other waves) is reduced inside the medium.

### 2.4.2 Lossy Dielectric Medium

The current term in the wave equation can be included to represent the power loss during wave propagation. In this case, the induced current is related to the electric field by $\mathbf{J} = \sigma \mathbf{E}$, where $\sigma$ is the conductivity. If the electric

field has a time dependence $e^{j\omega t}$, the induced current density also has the same time dependence. Therefore, this current effect is included in the electric susceptibility as follows. From (2.2),

$$\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\mathbf{J}}{j\omega} + \mathbf{D}\right) = \frac{\partial}{\partial t}(\varepsilon \mathbf{E}), \tag{2.26}$$

with the dielectric constant $\varepsilon$.

In general, $\varepsilon(r, \omega)$ is complex. Its real and imaginary parts correspond to the refractive index $n$ and the absorption coefficient $\alpha$ by the definition [202]

$$\varepsilon = (n + j\alpha c/2\omega)^2, \tag{2.27}$$

By using (2.26) and (2.27), $n$ and $\alpha$ are related to $\chi$ as

$$n = (1 + \mathrm{Re}\{\chi\})^{1/2}, \tag{2.28}$$

$$\alpha = (\omega/nc)\mathrm{Im}\{\chi\}, \tag{2.29}$$

where Re and Im stand for the real and imaginary parts, respectively. Both $n$ and $\alpha$ are frequency dependent. The imaginary part of the refractive index represents either losses (if negative) or gain (if positive). The frequency dependence of $n$ is referred to as chromatic dispersion. In case of losses present in medium, the radiation exponentially decays as shown in Fig. 2.4.

If $\mathbf{D}$ is substituted in (2.2) from (2.5), the wave equation for the electric field is reduced to

$$\nabla \mathbf{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = \mu_0 \frac{\partial^2}{\partial t^2}\mathbf{P}. \tag{2.30}$$

Using the Fourier transform of $\mathbf{E}(\mathbf{r}, t)$ formula (2.31)

$$\tilde{E}(\mathbf{r}, \omega) = \int \mathbf{E}(\mathbf{r}, t) \exp(j\omega t) \mathrm{d}t \tag{2.31}$$

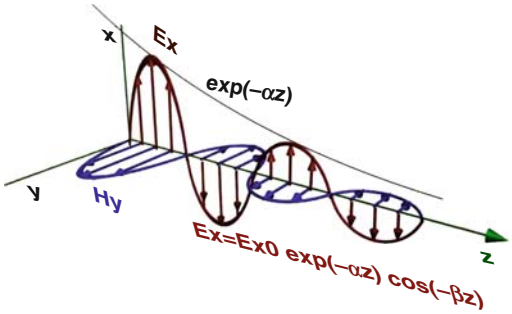


**Fig. 2.4.** Wave in lossy dielectric medium

and similar relation for $\mathbf{P}(\mathbf{r},t)$ together with (2.7), one can write wave equation (2.15) in the frequency domain

$$\nabla \times \nabla \times \tilde{E} = -\varepsilon(\mathbf{r},\omega)\frac{\omega^2}{c^2}\tilde{E}, \tag{2.32}$$

with the frequency-dependent permittivity

$$\varepsilon(\mathbf{r},\omega) = 1 + \tilde{\chi}(\mathbf{r},\omega) \tag{2.33}$$

and $\tilde{\chi}(\mathbf{r},\omega)$ is the temporal Fourier transform of $\chi(\mathbf{r},t)$. In term of frequencies the wave equation can be obtained as

$$\nabla^2\tilde{E} + n^2(\omega)k_0^2\tilde{E} = 0, \tag{2.34}$$

where the free-space wave number $k_0$ is defined as

$$k_0 = \omega/c = 2\pi/\lambda_0 \tag{2.35}$$

and $\lambda_0$ is the vacuum wavelength of the optical field oscillating at the frequency $\omega$. Equation (2.34) is solved next to obtain the optical modes.

## 2.5 Group Velocity

The concept of group velocity is important for understanding lightwave propagation and quantifying fiber dispersion. In general, there are two different velocities: group velocity and phase velocity. To see the meanings of these velocities, let us first consider a traveling wave of the following form:

$$E(t,z) = E_0\ \cos(\omega_0 t - \beta z). \tag{2.36}$$

Its Fourier transform with respect to time is

$$E(\omega,z) = E_0\pi[\delta(\omega-\omega_0)e^{-j\beta z} + \delta(\omega+\omega_0)e^{j\beta z}]. \tag{2.37}$$

This means that a plane wave has a constant frequency. This constant frequency wave is called a monochromatic wave and is also called a continuous wave (CW). In this case, the phase velocity is defined as

$$v_p = \frac{\omega_0}{\beta}. \tag{2.38}$$

Thus, phase velocity is the velocity of a constant phase plane that moves in the propagation direction.

For the plane wave considered, its amplitude or power is constant over the entire space, time- and position-independent. In other words, one cannot tell

"where" the wave is. Actually, it is everywhere. On the other hand, for two plane waves with a little difference in frequency and wave number,

$$E(t,z) = E_0 \cos(\omega_1 t - \beta_1 z) + E_0 \cos(\omega_2 t - \beta_2 z), \tag{2.39}$$

after trigonometric transformations, takes form:

$$E(t,z) = 2E_0 \cos(\Delta\omega t/2 - \Delta\beta z)\cos(\bar{\omega} t - \bar{\beta} z), \tag{2.40}$$

where $\Delta\omega = \omega_2 - \omega_1$ and $\Delta\beta = \beta_2 - \beta_1$ are assumed to be small, compared to $\bar{\omega} = (\omega_1 + \omega_2)/2$ and $\bar{\beta} = (\beta_1 + \beta_2)/2$, respectively. Thus, the envelope of the combined wave has a velocity

$$v_{\mathrm{g}} = \frac{\Delta\omega}{\Delta\beta}. \tag{2.41}$$

This is called the group velocity. As the derivation shows, it means the propagation velocity of energy.

The concept of group velocity can be generalized from the two monochromatic waves to a "Wave packet". This form gives a possibility to represent the group velocity like

$$v_{\mathrm{g}} = \frac{\partial\omega}{\partial\beta}. \tag{2.42}$$

The above presentation shows that each component at a different frequency travels at the same speed of $v_{\mathrm{g}}$. In other words, the wave packet or its amplitude travels at the group velocity $v_{\mathrm{g}}$.

## 2.6 Summary

1. Electromagnetic waves are described by a wave equation arising from Maxwell's equations. Electromagnetic waves provide energy transport described by the Poynting vector.
2. In general, in a homogeneous linear and isotropic dielectric medium, the polarization is aligned with and proportional to the electric field **E**. In an anisotropic material, the polarization and the field are not necessarily in the same direction.
3. The phase velocity is the velocity of constant phase plane.
4. The group velocity is the velocity of the enegry propagation.
5. A wave vector is a vector which specifies the wavenumber and direction of propagation for a wave. The magnitude of the wave vector indicates the wavenumber. The orientation of the wave vector indicates the direction of wave propagation. Wavenumber is a measure of the number of repeating units of a propagating wave per unit of space and is measured in $\mathrm{m}^{-1}$.
6. The refractive index of a medium is a measure for how much the speed of light (or other waves) is reduced inside the medium.

## 2.7 Problems and Questions

**Problem 2.1.** What is permittivity of the medium?

**Problem 2.2.** How the refractive index is connected to permittivity?

**Problem 2.3.** Derive the wave equation from the Maxwell's equations in vacuum.

**Problem 2.4.** Derive the frequency domain Helmholtz equation in dielectric medium.

**Problem 2.5.** Show that Maxwell's equation $\nabla \times \mathbf{E} = -\partial B/\partial t$ and $\nabla \cdot \mathbf{E} = 0$ are consistent, i.e., they do not contradict each other.

**Problem 2.6.** Explain terms "plane wave" and "polarization".

**Problem 2.7.** Search out the magnetic strength $H$ for nonuniform plane wave in a medium with parameters $\varepsilon$ and $\mu$, if the electric strength is given by: $E_y = E_0 \ \exp(i(\omega t - k_z \cdot z) - k_x \cdot x)$, $E_x = E_y = 0$.